Models for Inexact Reasoning

Reasoning with Subjective Pseudo-Probabilities: The PROSPECTOR Approach

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The PROSPECTOR System

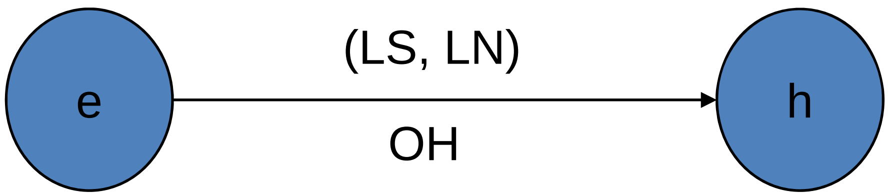
- Developed by Duda and Hart during the 70s at SRI International
- Intended user is an exploration geologist in the early stages of investigating a possibly drilling site
- Successfully used in practice:
  - Helped to discover an important deposit of Molybdenum worth more than $100M in the State of Washington (USA)

The PROSPECTOR Approach

- Rule-based system (backwards chaining)
- Use of an inference tree to support the reasoning process (similarly to MYCIN)
- PROSPECTOR uses probabilities to represent uncertain knowledge
  - "A priori"
    - Independent of the case
    - Independent of the problem to be solved (the hypothesis)
  - "A posteriori"
    - Evidence for a concrete case (data dependent)

Sufficiency, Necessity and Credibility Factors

- PROSPECTOR uses three measures to represent the conditional probabilities involved in an implication:
  - Sufficiency (LS_e)
  - Necessity (LN_e)
  - Credibility (O)
- All these factors are provided by experts



Credibility Factor

- Given a hypothesis *h,* PROSPECTOR is targeted to analyze the evolution of the ratio between:
  - The probability of *h* to hold
  - The probability of *h* to be false
- This ratio is called the Credibility Factor

$$O(h) = \frac{P(h)}{P(\bar{h})} = \frac{P(h)}{1 - P(h)}$$

Evolution of the Credibility Factor

- Evolution of "a priori" CFs due to evidence
  - In the case of *e* to be true

$$O(h \mid e) = LS_e \cdot O(h)$$

  - In the case of *e* to be false

$$O(h \mid \overline{e}) = LN_e \cdot O(h)$$

  - In the case of having n independent evidences over the hypothesis h

$$O(h \mid e_1, e_2, \overline{e}_3, \ldots, e_n) = LS_{e_1} \cdot LS_{e_2} \cdot LN_{e_3} \cdot \ldots \cdot LS_{e_n} \cdot O(h)$$

Sufficiency Factor

- Measures the extent to which the antecedent is sufficient for the consequent to hold
  - Degree to which if the premises are true then the hypothesis is also true
- The higher the better (ideally ∞)

$$LS_e = \frac{P(e \mid h)}{P(e \mid \bar{h})}$$

Necessity Factor

- Measures the extent to which the antecedent is necessary for the consequent to hold
  - Degree to which if the consequent is true then the premises are also true
- The smaller the better (ideally 0)

$$LN_e = \frac{P(\overline{e} \mid h)}{P(\overline{e} \mid \overline{h})}$$

Interpretation of the Degree of Sufficiency

LS	Effects over the implication [*e* → *h*]
0	*h* is false when *e* holds or $\bar{e}$ is necessary for *h* to hold
0 < LS << 1	*e* is not favorable for *h* to hold
1	e has no effect over *h*
1 << LS	*e* is favorable for *h* to hold
∞	*e* is sufficient for *h* to hold or if *e* holds then *h* holds

Interpretation of the Degree of Necessity

LN	Effects over the implication [$e \rightarrow h$]
0	*h* is false when *e* does not hold or *e* is necessary for *h* to hold
0 < LN << 1	$\bar{e}$ is not favorable for *h* to hold
1	$\bar{e}$ has no effect over *h*
1 << LN	$\bar{e}$ is favorable for *h* to hold
∞	$\bar{e}$ is sufficient for *h* to hold

Inference Process

- Given a concrete case E including:
  - Observed probabilities $P(e_i|E)$ for facts $e_1,\ldots,e_i$
  - A target hypothesis h
- Aim: Calculate P(h|E)
- Applying the total probability theorem:

$$P(h\,|\,E) = P(h\,|\,e)\cdot P(e\,|\,E) + P(h\,|\,\overline{e})\cdot P(\overline{e}\,|\,E)$$

$$\updownarrow$$

$$P(h\,|\,E) = P(h\,|\,e)\cdot P(e\,|\,E) + P(h\,|\,\overline{e})\cdot (1 - P(e\,|\,E))$$

Inference Process

- The latter is the equation of a "line" satisfying the following conditions:

$$[P(e|E)=0] \leftrightarrow [P(h|E)=P(h|\overline{e})]$$

$$[P(e|E)=P(e)] \leftrightarrow [P(h|E)=P(h)]$$

$$[P(e|E)=1] \leftrightarrow [P(h|E)=P(h|e)]$$

- This "line" is not a line (but a piecewise linear interpolation) **why??**
  - The pair [P(e), P(h)] was provided by experts
  - It does not satisfy any mathematical conditions

Inference Process

- We can calculate $P_R(h|E)$ (due a single rule R) given $P(e|E)$ using piecewise linear interpolation
- First, we calculate $P(h|e)$ and $P(h|\bar{e})$ as follows:

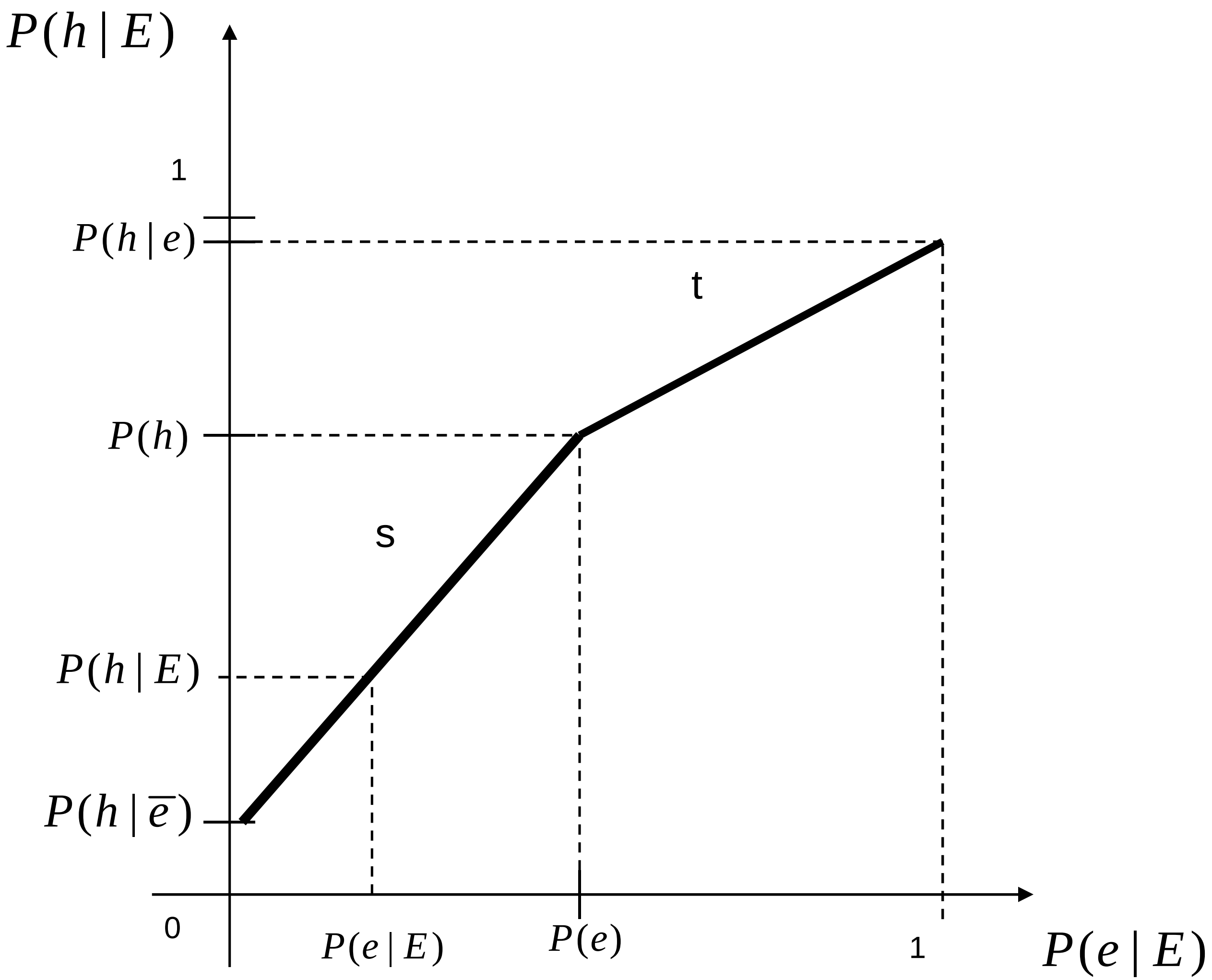
$$P(h|e) = \frac{O(h|e)}{1 + O(h|e)}$$

$$P(h|\overline{e}) = \frac{O(h|\overline{e})}{1 + O(h|\overline{e})}$$

Inference Process

- Then, we obtain the equation for line s or t depending on the following conditions:
  - If $0 < P(e|E) < P(e)$
    - s → [0, P(h|ē)], [p(e), p(h)]
  - If $P(e) < P(e|E) < 1$
    - t → [p(e), p(h)], [1, p(h|e)]
- After that, we calculate $P_R(h|E)$ using the corresponding equation and the value $P(e|E)$

Inference Process



Inference Process

- From $P_R(h|E)$ we calculate $O_R(h|E)$

$$O_R(h|E) = \frac{P_R(h|E)}{1 - P_R(h|E)}$$

- And from $O_R(h|E)$ we calculate L_R

$$L_R = \frac{O_R(h|E)}{O(h)}$$

Inference Process

- If several rules involve the same consequent, we accumulate the beliefs contributed by the different rules:

$$O(h \mid E) = L_{R_1} \cdot L_{R_2} \cdot \ldots \cdot L_{R_n} \cdot O(h)$$

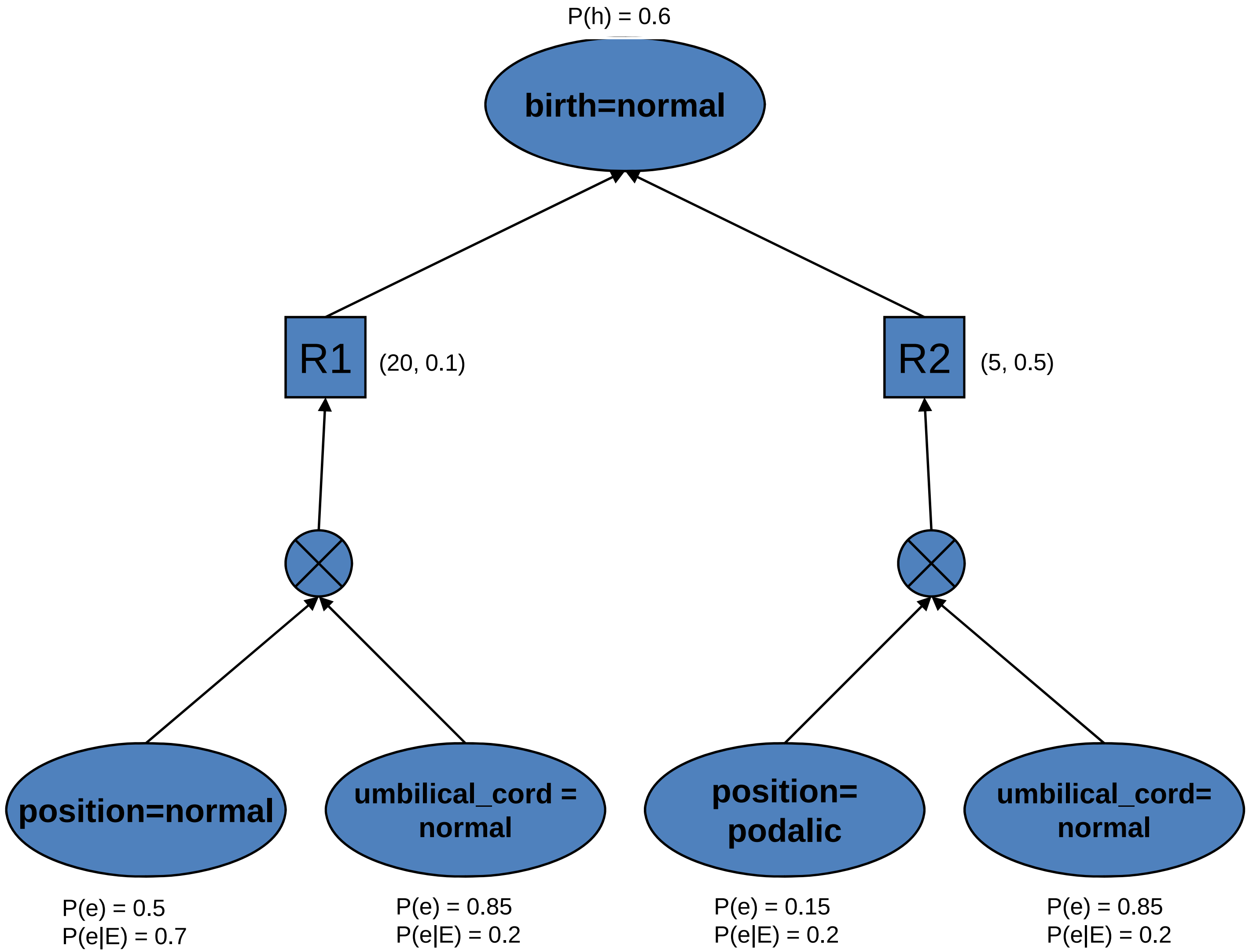
- Then, we calculate P(h|E) as follows:

$$P(h \mid E) = \frac{O(h \mid E)}{1 + O(h \mid E)}$$

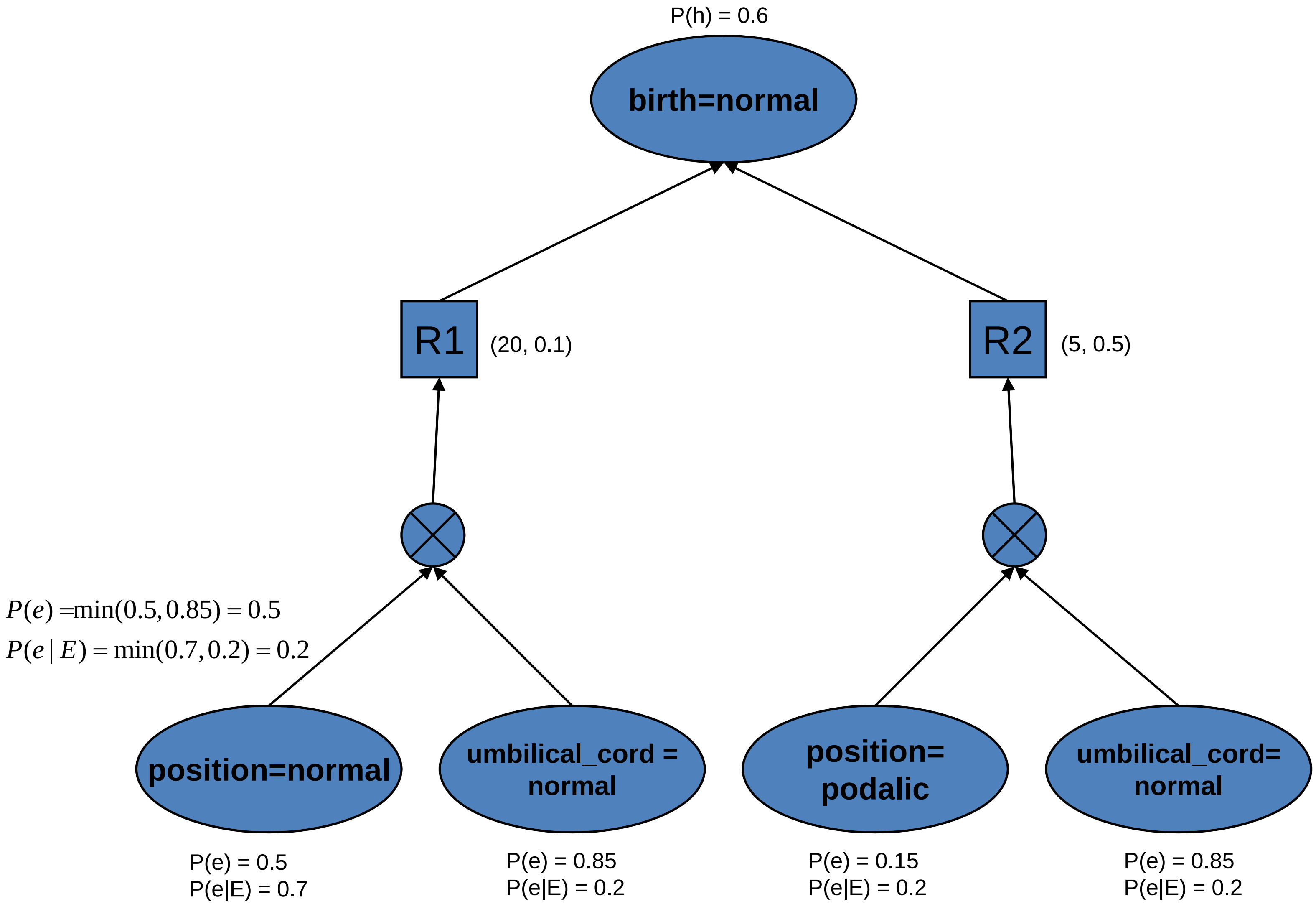
Inference Process: Example

- Rules:
  - R1: IF (position=normal) AND (umbilical_cord=normal) THEN (20, 0.1) (birth=normal)
  - R2: IF (position=podalic) AND (umbilical_cord=normal) THEN (5, 0.5) (birth=normal)
- "A priori" evidence
  - P(position=normal) = 0.5
  - P(position=podalic) = 0.15
  - P(umbilical_cord=normal) = 0.85
  - P(birth=normal) = 0.60
- A concrete case (E)
  - P(position=normal|E) = 0.70
  - P(position=podalic|E) = 0.20
  - P(umbilical_cord=normal|E) = 0.20
  - P(birth=normal|E)?

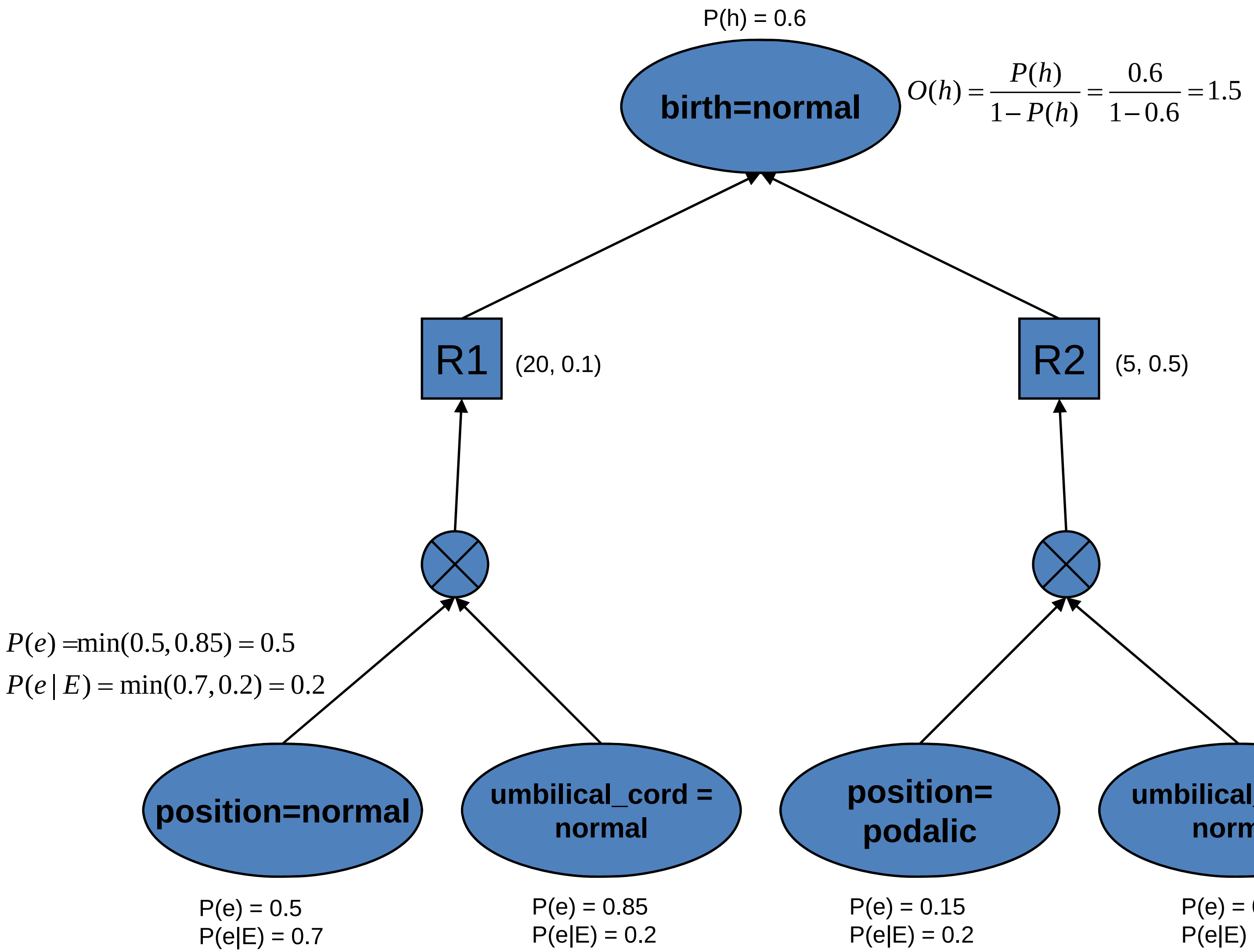
Example: Inference Tree



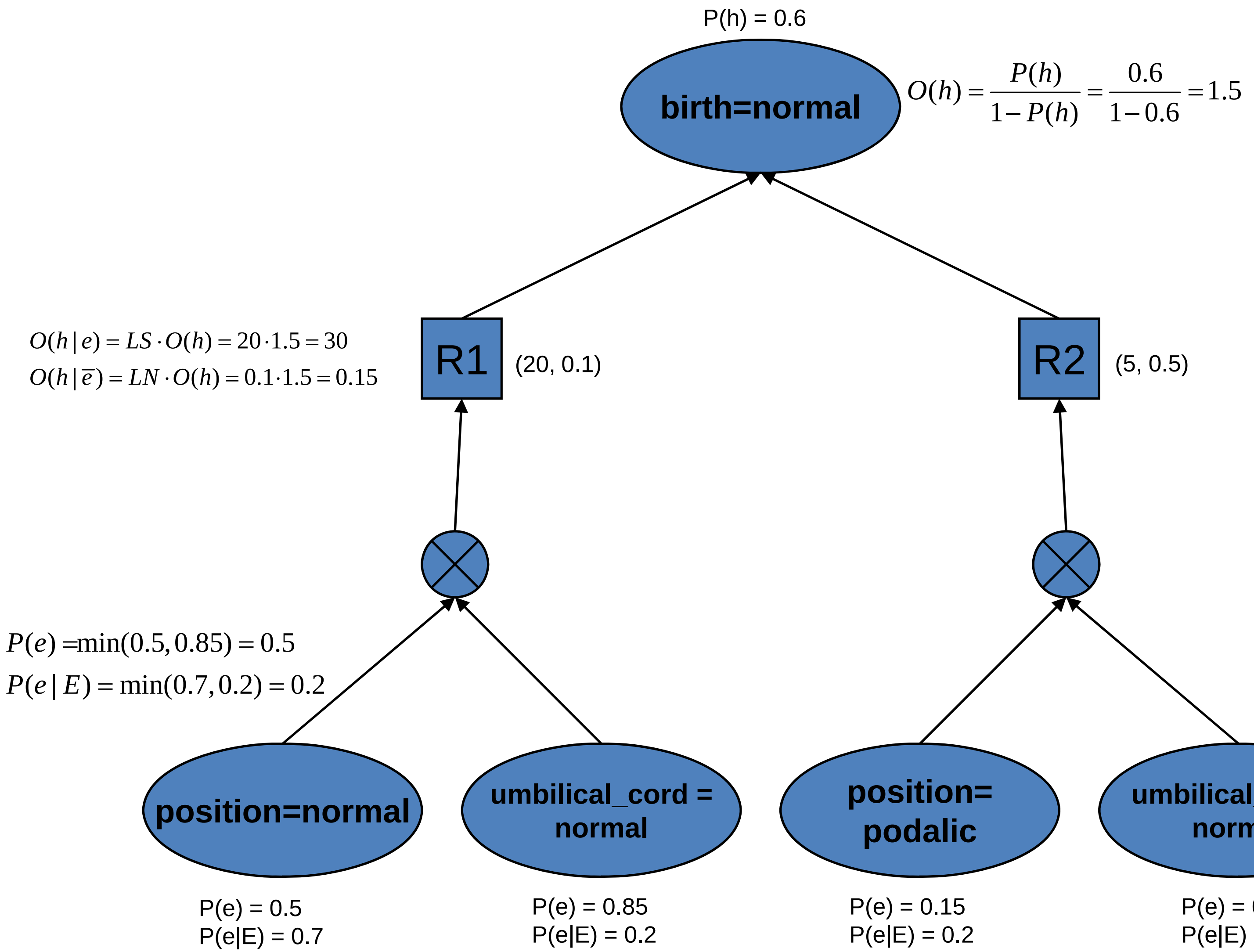
Step 1: Combination of “a priori” and “a posteriori” facts beliefs for rule R1



Step 2: Calculate the “a priori” credibility of the hypothesis



Step 3: Calculate O(h|e) and O(h|ē) for rule R1

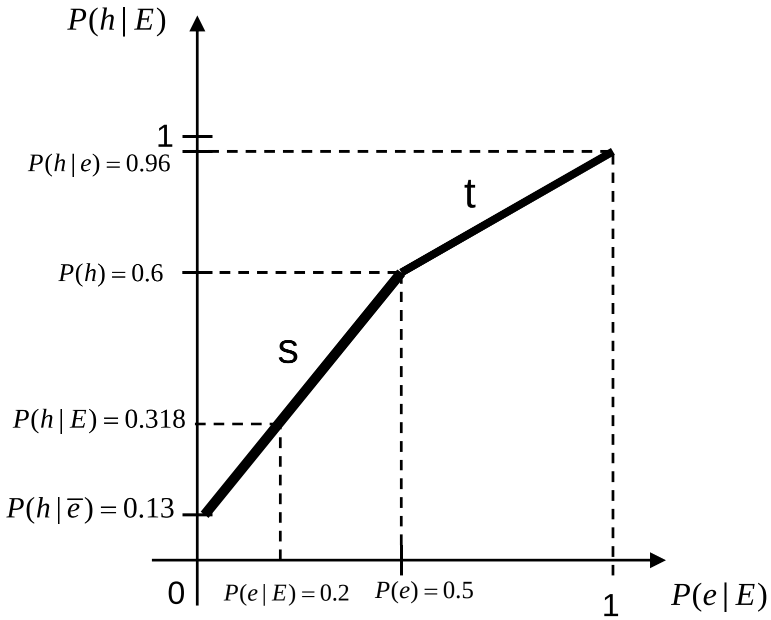


Step 4: Propagation of "a posteriori" evidence from facts to hypothesis (rule R1)

$$P(h|e) = \frac{O(h|e)}{1+O(h|e)} = \frac{30}{1+30} = 0.96$$

$$P(h|\bar{e}) = \frac{O(h|\bar{e})}{1+O(h|\bar{e})} = \frac{0.15}{1+0.15} = \frac{0.15}{1.15} = 0.13$$

P(e|E)	**P(h|E)**
0	P(h|ē)=0.13
P(e)=0.5	P(h)=0.6
1	P(h|e)=0.96

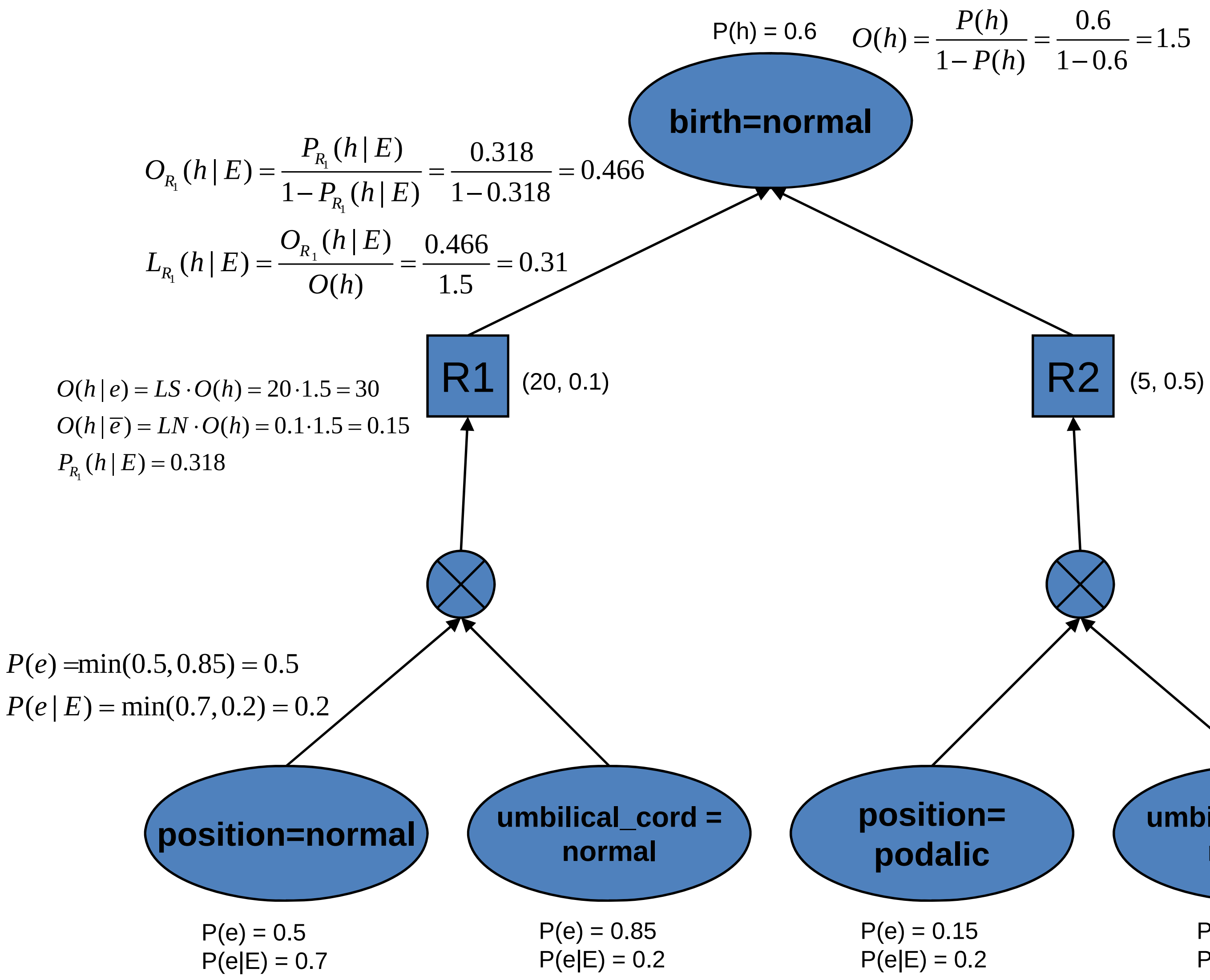


$$s: P(h|E) = 0.94 \cdot P(e|E) + 0.13$$

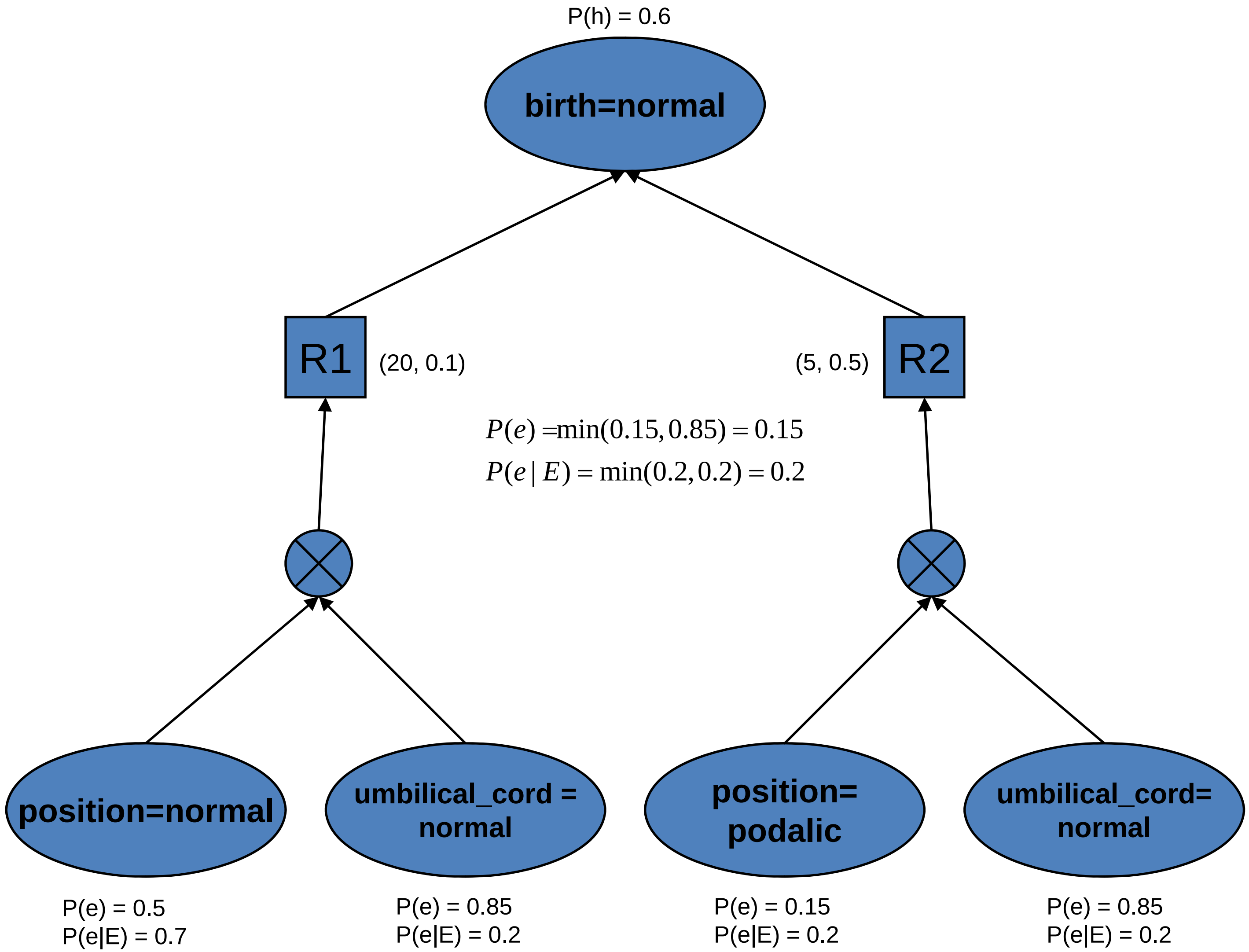
$$t: P(h|E) = 0.72 \cdot P(e|E) + 0.24$$

$$P(h|E) = 0.94 \cdot 0.2 + 0.13 = 0.318$$

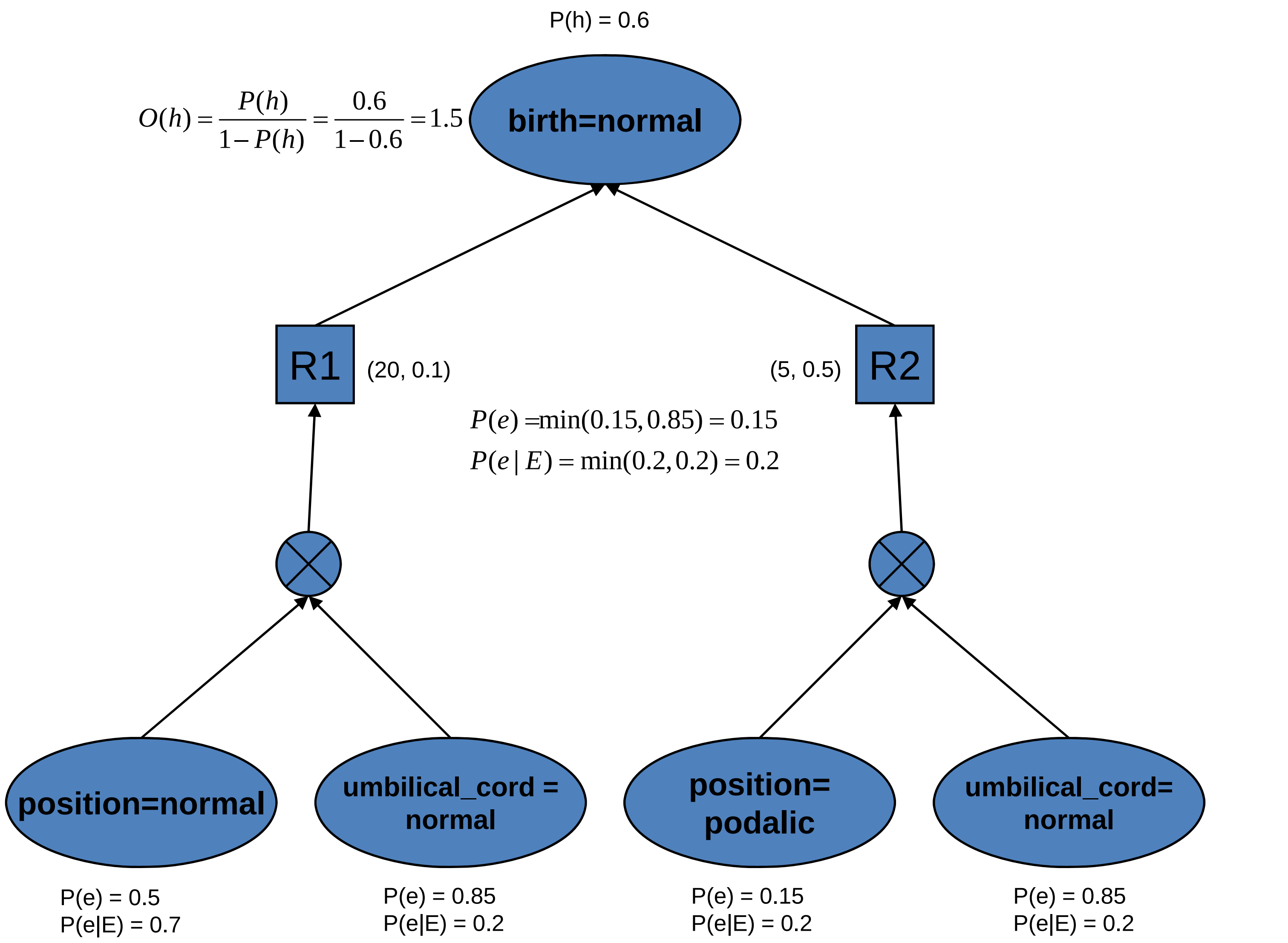
Step 4: Propagation of "a posteriori" evidence from facts to hypothesis (rule R1)



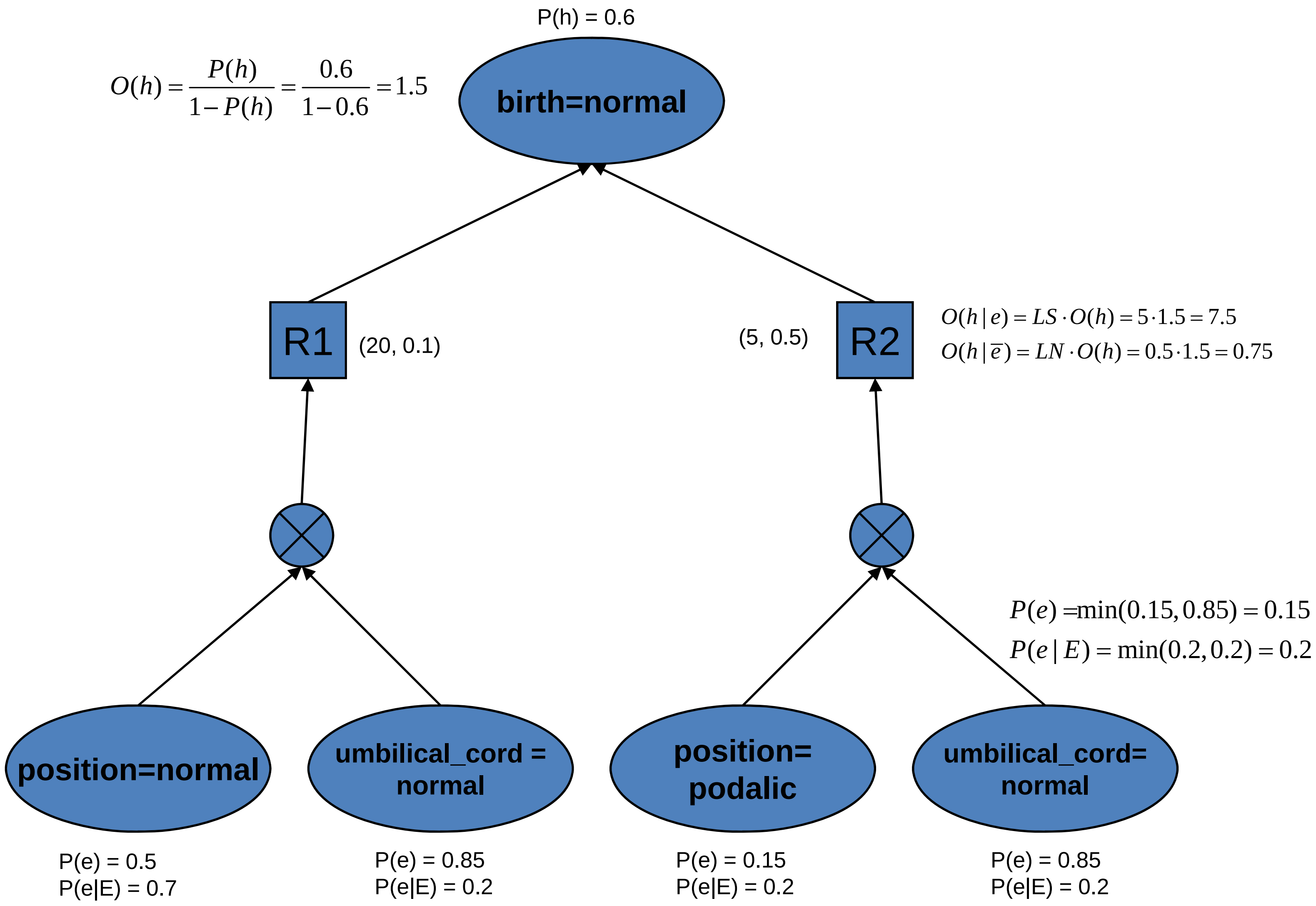
Step 1: Combination of “a priori” and “a posteriori” facts beliefs for rule R2



Step 2: Calculate the "a priori" credibility of the hypothesis



Step 3: Calculate O(h|e) and O(h|ē) for rule R2

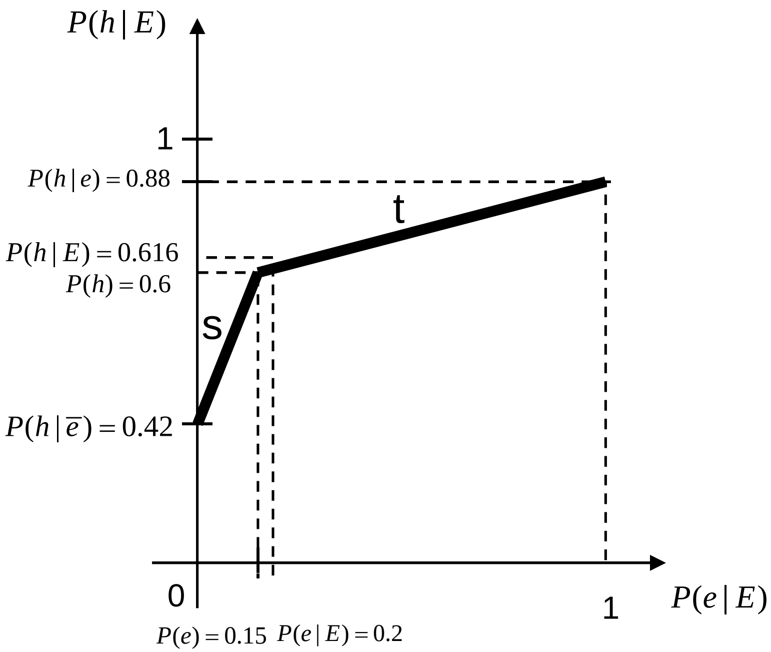


Step 4: Propagation of "a posteriori" evidence from facts to hypothesis (rule R2)

$$P(h|e)=\frac{O(h|e)}{1+O(h|e)}=\frac{7.5}{1+7.5}=0.88$$

$$P(h|\bar{e})=\frac{O(h|\bar{e})}{1+O(h|\bar{e})}=\frac{0.75}{1+0.75}=\frac{0.75}{1.75}=0.42$$

P(e|E)	P(h|E)
0	P(h|ē)=0.42
P(e)=0.15	P(h)=0.6
1	P(h|e)=0.88

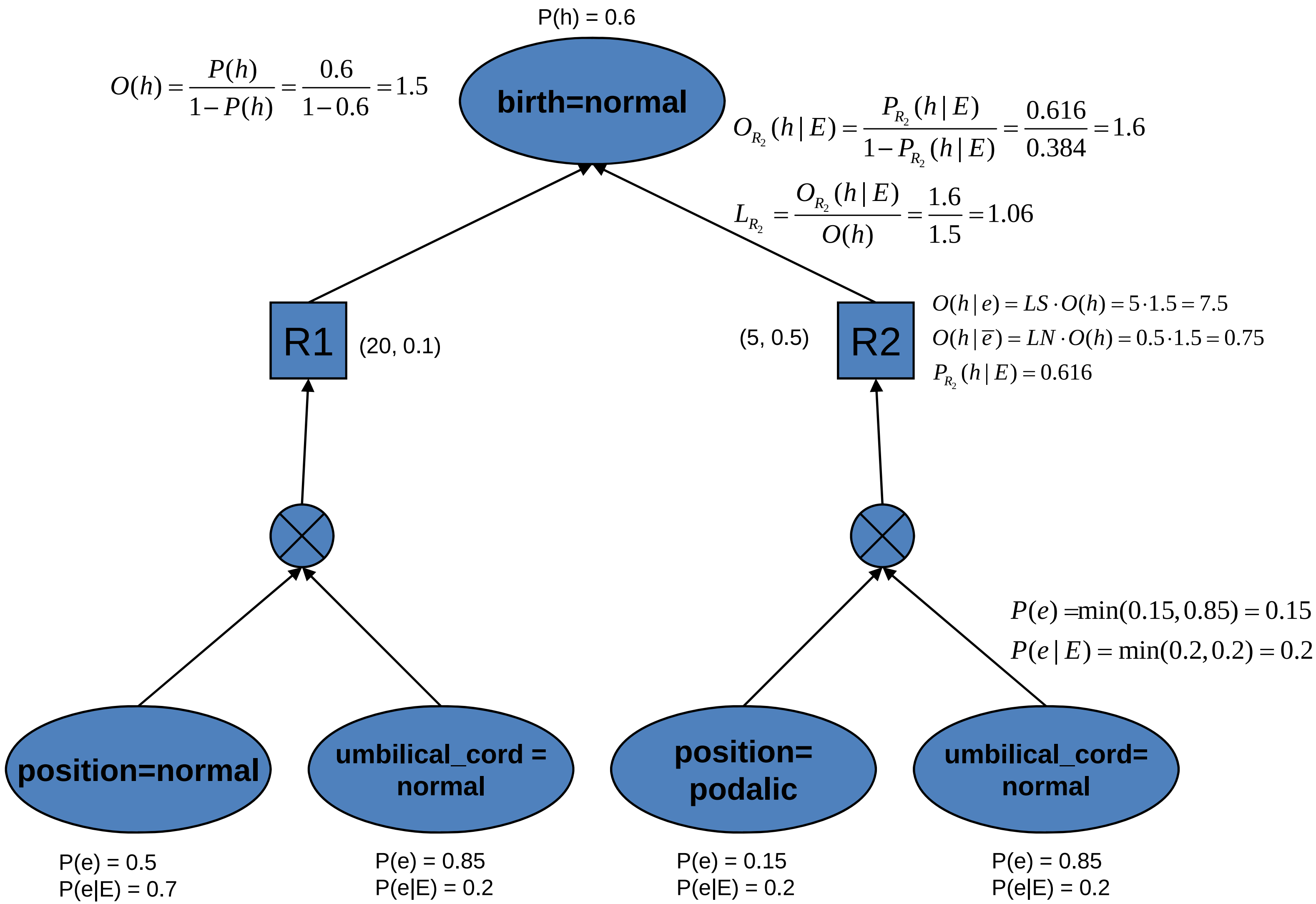


$$s: P(h|E)=1.2\cdot P(e|E)+0.42$$

$$t: P(h|E)=0.33\cdot P(e|E)+0.55$$

$$P(h|E)=0.33\cdot 0.2+0.55=0.616$$

Step 4: Propagation of "a posteriori" evidence from facts to hypothesis (rule R2)



Final Step: Accumulation of beliefs from multiple rules

$$O(h|E) = L_{R_1} L_{R_2} O(h) = 0.31 \cdot 1.06 \cdot 1.5 = 0.49$$

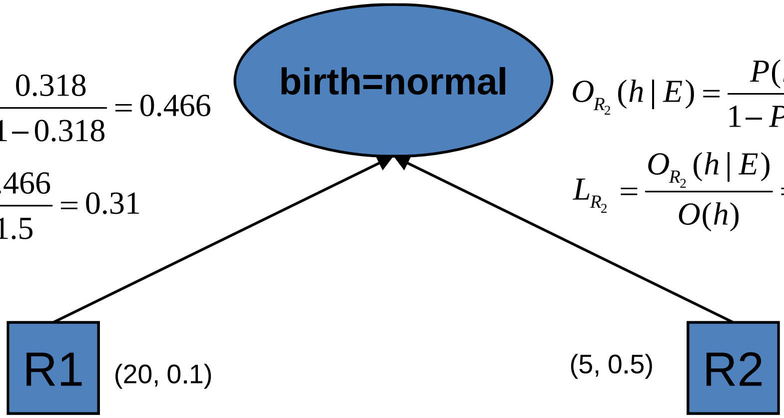
$$O(h) = \frac{P(h)}{1 - P(h)} = \frac{0.6}{1 - 0.6} = 1.5$$

$$O_{R_1}(h|E) = \frac{P(h|E)}{1 - P(h|E)} = \frac{0.318}{1 - 0.318} = 0.466$$

$$L_{R_1}(h|E) = \frac{O_{R_1}(h|E)}{O(h)} = \frac{0.466}{1.5} = 0.31$$

$$O_{R_2}(h|E) = \frac{P(h|E)}{1 - P(h|E)} = \frac{0.616}{0.384} = 1.6$$

$$L_{R_2} = \frac{O_{R_2}(h|E)}{O(h)} = \frac{1.6}{1.5} = 1.06$$



$$P(h|E) = \frac{O(h|E)}{1 + O(h|E)} = \frac{0.49}{1 + 0.49} = 0.33$$